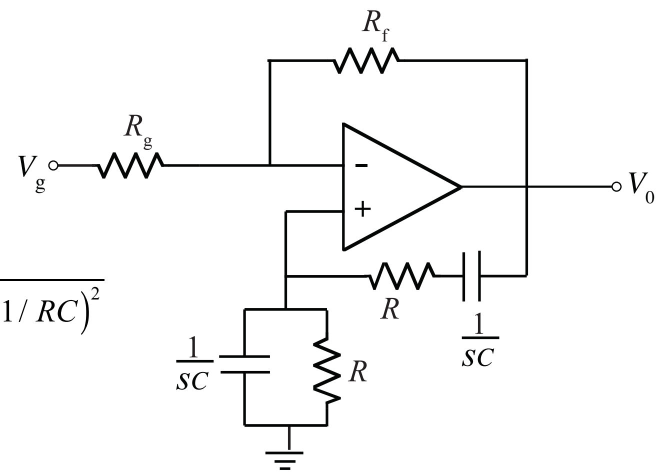
Salen-Key Oscillator



Voltage divider at the + pin:

$$V_+(s)=V_0(s)\frac{1/(sC+1/R)}{R+\frac{1}{sC}+\left[1/(sC+1/R)\right]}=V_0(s)\frac{s/RC}{s^2+3s/RC+(1/RC)^2}$$

KCL at the – pin and noting that $V_+ = V_-$:

$$\frac{V_- - V_g}{R_g}+\frac{V_- - V_0}{R_f}=0$$

After *a lot* of manipulations, these equations yield:

$$\boxed{V_0(s) = -\frac{R_f}{R_g}V_g(s)\,\frac{s^2+\frac{3}{RC}s+\left(\frac{1}{RC}\right)^2}{s^2+\left(\frac{3-G}{RC}\right)s+\left(\frac{1}{RC}\right)^2}}$$

Where $G=1+R_f/R_g$ is the dc gain of the op amp circuit.

We know of course that it's the denominator that determines the nature of the transient response.

Comparing this denominator, the standard form: $s^2+\left(\frac{3-G}{RC}\right)s+\left(\frac{1}{RC}\right)^2 = s^2+2\varsigma\omega_0 s + \omega_0{}^2$,

we find: $\varsigma=\frac{1}{2}(3-G)$ and $\omega_o=1/RC$. From this, we see that the transient response of this circuit will be of the form: $e^{-\omega_0\varsigma t}\cos(\omega_0\sqrt{1-\varsigma^2}\,t)$.

Clearly, if, $G=\left(1+\frac{R_f}{R_g}\right)=3$, $\zeta=0$, and the output will NOT decay with time so the circuit that produces a sinusoid of frequency $f=\frac{\omega_o}{2\pi}=\frac{1}{2\pi RC}$.